Full-wave Bridge Rectifier Analysis

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This paper develops approximate equations for designing or analyzing a full-wave bridge rectifier peak-detector circuit. This circuit is commonly used in AC to DC converters, and consists of only six circuit elements. Even so, the circuit analysis for the full-bridge rectifier circuit is not as simple as first impressions might lead one to believe.

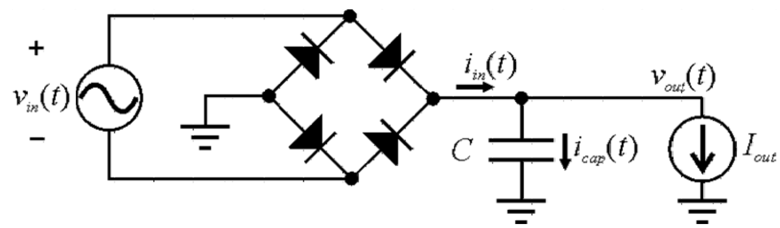


Figure 1. Full bridge rectifier circuit

Figure 1 shows the schematic for the full-wave bridge rectifier. The four-diode bridge converts both polarities of the input waveform into positive voltage at the output. The capacitor connected to the output node acts as a charge reservoir, which smoothes the output voltage and makes the circuit more like constant DC voltage source.

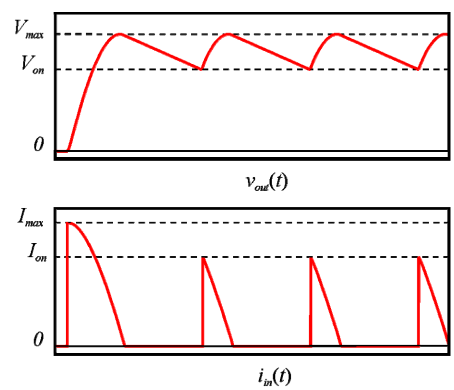


Figure 2. Output voltage and input current

As shown in Figure 2, the output voltage waveform has a ripple, which is dependent on the capacitor size and the amount of output current. The input current, $i_{in}(t)$, consists of a trail of triangular pulses, which occur when the diodes turn on. To analyze the full-wave bridge rectifier, we will analyze one conduction cycle for the circuit. Each conduction cycle lasts half a period, and consists of two phases: the **off phase**, when no current is flowing through the diodes to the output; and the **on phase**, when current is being supplied through the diodes to the output. Figure 3 shows the output voltage and capacitor current plots during one conduction cycle.

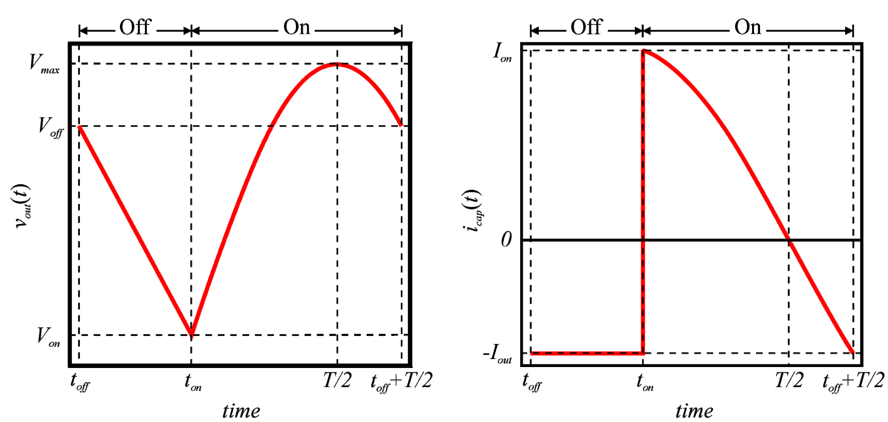


Figure 3. Output voltage and capacitor current for the full-wave rectifier during one conduction cycle

To begin the analysis, let

$$v_{in}(t) = -V_p \cos(\omega\, t)$$

This equation for $v_{in}(t)$ is chosen so that the conduction cycle starts out in the **off phase**. V_p is the peak amplitude of the input signal. During the **on phase**, the output voltage is equal to $v_{in}(t)$ minus two diode voltage drops. The **on phase** starts at time t_{on} and ends at t_{off} + T/2, which is the beginning of the next conduction cycle.

$$v_{out}(t,\text{on}) = -V_p \cos(\omega\, t) - 2\, V_D$$

At the beginning of the **off phase**, the diodes turn off, and the capacitor supplies all of the output current.

$$i_{cap}(t,\text{off}) = -I_{out}$$

The value t_{off} is the beginning of the **off phase**. To find t_{off}, we find the time at which the diodes turn off. This occurs at the time the slopes of $v_{off}(t,\text{off})$ for the current cycle and $v_{on}(t,\text{on})$ for the previous cycle are equal.

$$\frac{d}{dt} v_{out}(t,\text{off}) = \frac{d}{dt} v_{out}(t - T/2, \text{on})$$

$$\frac{d}{dt}\left[V_{off} - \frac{I_{out} \cdot (t - t_{off})}{C}\right] = \frac{d}{dt}\left[V_p \cos(\omega\, t) - 2\, V_D\right]$$

$$-\frac{I_{out}}{C} = -\omega\, V_p \sin(\omega\, t_{off})$$

$$t_{off} = \frac{1}{\omega} \sin^{-1}\left(\frac{I_{out}}{\omega\, C\, V_p}\right)$$

To simplify calculations, we use the trigonometric identity $\sin^{-1}(x) \cong x$.

$$t_{off} \cong \frac{I_{out}}{\omega^2\, C\, V_p}$$

We find V_{off} by plugging t_{off} + T/2 into $v_{out}(t,\text{off})$.

$$V_{off} = v_{out}(t_{off} + T/2, \text{off}) = -v_{out}(t_{off}, \text{off}) = V_p \cos\left(\frac{I_{out}}{\omega\, C\, V_p}\right) - 2\, V_D$$

The discharging capacitor causes the output voltage to drop. V_{off} is the voltage at the beginning of the off phase. The output voltage declines linearly with time with a slope dependant on the output current and the capacitor value.

$$v_{out}(t,\text{off}) = V_{off} + \frac{1}{C}\int_{t_{off}}^{t} i_{cap}(x,\text{off})\, dx = V_{off} - \frac{I_{out} \cdot (t - t_{off})}{C}$$

Based on the previous equation, the output voltage can be made steadier by increasing the capacitor size. Although the circuit more resembles an ideal constant voltage source, the diode current during the **on phase** is higher, meaning higher power diodes are necessary. In the design of rectifier circuits, limits are usually placed on the amount of ripple allowed in the output voltage, so V_{on} (the minimum output voltage reached during each cycle) is usually specified for the circuit along with the output current I_{out}. The next step is to find the capacitor values that would provide the specified amount of ripple. We first solve for t_{on}, the time at the beginning of the **on phase**, at which time the output voltage has dropped off the specified amount.

$$V_{on} = v_{out}(t_{on}, \text{on})$$

$$V_{on} = -V_p \cos(\omega\, t_{on}) - 2\, V_D$$

$$t_{on} = \frac{1}{\omega} \cos^{-1}\left(-\frac{V_{on} + 2\, V_D}{V_p}\right)$$

During the first part of the **on phase**, the capacitor current is positive, and the output current is fed directly out of the bridge diodes. At the peak output voltage, the capacitor current reverses, and the capacitor and diodes both contribute to the output current. The current going into the capacitor during the **on phase** is

$$i_{cap}(t,\text{on}) = C \cdot \frac{d}{dt} v_{out}(t,\text{on}) = \omega\, C\, V_p \sin(\omega\, t)$$

We now have enough information to solve for the capacitor value. We solve for C by setting V_{on} equal to $v_{out}(t_{on},\text{off})$.

$$V_{on} = V_{off} - v_{out}(t_{on},\text{off})$$

$$V_{on} = V_{off} - \frac{I_{out} \cdot (t_{on} - t_{off})}{C}$$

$$V_{on} = V_p \cos\left(\frac{I_{out}}{\omega\, C\, V_p}\right) - 2\, V_D - \frac{I_{out}}{C} \cdot \left(t_{on} - \frac{I_{out}}{\omega^2\, C\, V_p}\right)$$

We will use the third-order Taylor series polynomial to approximate cosine: $\cos(x) \cong 1 - x^2$.

$$V_{on} = V_p \left(1 - \frac{I_{out}^2}{\omega^2\, C^2\, V_p^2}\right) - 2\, V_D - \frac{I_{out}}{C} \cdot \left(t_{on} - \frac{I_{out}}{\omega^2\, C\, V_p}\right)$$

$$C \cdot (V_{on} - V_p + 2\, V_D) = -I_{out}\, t_{on} - \frac{I_{out}^2}{\omega^2\, C\, V_p} + \frac{I_{out}^2}{\omega^2\, C\, V_p}$$

$$C = -\frac{I_{out}\, t_{on}}{V_{on} - V_p + 2\, V_D}$$

The equation above is a rough approximation for the output capacitor, and will give higher capacitor values than needed. The lower the value of V_{on} is, the higher the error from the approximation.

The selection of rectifier diodes must take into account the initial surge current, I_{max}. This current is a result of the capacitor having no charge stored at the beginning of the first conduction cycle. The initial surge current is

$$I_{max} = \omega\, C\, V_p + I_{out}$$

I_{on} is the maximum current during the conduction cycle, which is at time t_{on}.

$$I_{on} = \omega\, C\, V_p \sin(\omega\, t_{on}) + I_{out}$$

The root mean square (rms) diode current is useful, since it is used for calculating the power dissipation in the diodes. The diode current is the same as the input current $i_{in}(t)$, except for that each diode turns on every other cycle. To find the rms current, we integrate the square of the input current over the **on phase**, find the average square current over a two-cycle interval by dividing by T, then take the square root of the result.

$$I_{D,\text{rms}} = \sqrt{\frac{\int_{\text{on}} i_{in}(t,on)^2\, dt}{T}} = \sqrt{\frac{\int_{t_{on}}^{t_{off}+T/2} \left[I_{out} + \omega\, C\, V_p \sin(\omega\, t)\right]^2\, dt}{T}}$$

We will work with the integral part of the equation.

$$\int_{t_{on}}^{t_{off}+T/2} \left[I_{out} + \omega\, C\, V_p \sin(\omega\, t)\right]^2\, dt$$

$$= \int_{t_{on}}^{t_{off}+T/2} \left[I_{out}^2 + 2\, \omega\, C\, V_p\, I_{out} \sin(\omega\, t) + \omega^2\, C^2\, V_p^2 \sin(\omega\, t)\right] dt$$

$$= \left\{ I_{out}^2\, t - 2\, C\, V_p\, I_{out} \cos(\omega\, t) + \frac{1}{2} \omega\, C^2\, V_p^2 \left[\omega\, t - \sin(\omega\, t) \cos(\omega\, t) \right] \right\} \Bigg|_{t=t_{on}}^{t_{off}+T/2}$$

$$= \left[t \cdot \left(I_{out}^2 + \frac{1}{2} \omega^2\, C^2\, V_p^2 \right) - 2\, C\, V_p\, I_{out} \cos(\omega\, t) - \frac{1}{2} \omega\, C^2\, V_p^2 \sin(\omega\, t) \cos(\omega\, t) \right] \Bigg|_{t=t_{on}}^{t_{off}+T/2}$$

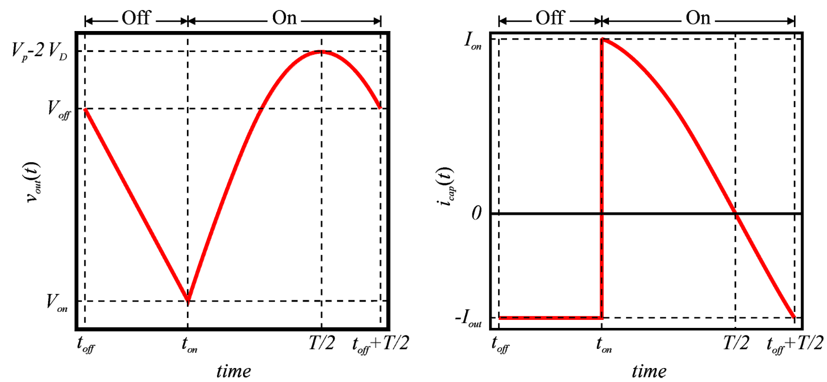
$$= \left(t_{off} + \frac{T}{2} - t_{on} \right) \cdot \left(I_{out}^2 + \frac{1}{2} \omega^2\, C^2\, V_p^2 \right) - 2\, C\, V_p\, I_{out} \left[\cos(\omega\, t_{on}) + \cos(\omega\, t_{off}) \right]$$
$$- \frac{1}{2} \omega\, C^2\, V_p^2 \left[\sin(\omega\, t_{on}) \cos(\omega\, t_{on}) - \sin(\omega\, t_{off}) \cos(\omega\, t_{off}) \right]$$

We use the trigonometric identity $\sin(x)\cos(x) - \sin(y)\cos(y) = \dfrac{\sin(2\,x) - \sin(2\,y)}{2}$ to simplify this equation.

$$= \left(t_{off} + \frac{T}{2} - t_{on} \right) \cdot \left(I_{out}^2 + \frac{1}{2} \omega^2\, C^2\, V_p^2 \right) - 2\, C\, V_p\, I_{out} \left[\cos(\omega\, t_{on}) + \cos(\omega\, t_{off}) \right] - \frac{1}{4} \omega\, C^2\, V_p^2 \left[\sin(2\, \omega\, t_{on}) - \sin(2\, \omega\, t_{off}) \right]$$

The final equation for RMS current is

$$I_{D,\mathrm{rms}} = \sqrt{\frac{\left(t_{off} + \frac{T}{2} - t_{on} \right) \cdot \left(I_{out}^2 + \frac{1}{2} \omega^2\, C^2\, V_p^2 \right) - 2\, C\, V_p\, I_{out} \left[\cos(\omega\, t_{on}) + \cos(\omega\, t_{off}) \right] - \frac{1}{4} \omega\, C^2\, V_p^2 \left[\sin(2\, \omega\, t_{on}) - \sin(2\, \omega\, t_{off}) \right]}{T}}$$



Output voltage and capacitor current for the full-wave rectifier during one conduction cycle

Summary of equations for the full-wave bridge rectifier

Variable	**Equation**
Off phase	$t_{off} + k \cdot T/2 \le t < t_{on} + k \cdot T/2$
On phase	$t_{on} + k \cdot T/2 \le t \le t_{off} + (k+1) \cdot T/2$
$v_{out}(t)$	$\begin{cases} V_{off} - I_s \cdot (t - t_{off})/C & \text{Off phase} \\ -V_p \cos(\omega t) - 2\,V_D & \text{On phase} \end{cases}$
$i_{cap}(t)$	$\begin{cases} -I_{out} & \text{Off phase} \\ \omega\, C\, V_p \sin(\omega t) & \text{On phase} \end{cases}$
$i_{in}(t)$	$i_{cap}(t) + I_{out}$
t_{on}	$\frac{1}{\omega} \cos^{-1}\left(-\frac{V_{on} + 2\,V_D}{V_p} \right)$
C	$-\frac{I_{out}\, t_{on}}{V_{on} - V_p + 2\,V_D}$
t_{off}	$\frac{I_{out}}{\omega^2\, C\, V_p}$
V_{off}	$V_p \cos\left(\frac{I_{out}}{\omega\, C\, V_p} \right) - 2\,V_D$
V_{max}	$V_p - 2\,V_D$
I_{on}	$\omega\, C\, V_p \sin(\omega\, t_{on}) + I_{out}$
I_{max}	$\omega\, C\, V_p + I_{out}$
$I_{D,\mathrm{rms}}$	$\sqrt{\frac{\left(t_{off} + \frac{T}{2} - t_{on}\right) \cdot \left(I_{out}^2 + \frac{1}{2}\omega^2\, C^2\, V_p^2\right) - 2\, C\, V_p\, I_{out} \left[\cos(\omega\, t_{on}) + \cos(\omega\, t_{off})\right] - \frac{1}{4}\omega\, C^2\, V_p^2 \left[\sin(2\,\omega\, t_{on}) - \sin(2\,\omega\, t_{off})\right]}{T}}$